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# Global Optimization Techniques for Fluid Flow and Propulsion Devices

Wei SHYY, Nilay PAPILA, Raj VAIDYANATHAN,
Kevin TUCKER*, Lisa GRIFFIN*& Dan DORNEY*

• Aerospace Engineering, Mechanics & Engineering Science Department •
University of Florida • Gainesville • FL •

*NASA Marshall Space Flight Center • Huntsville • AL•

In Collaboration with Frank HUBER and Ken TRAN

# Why Global Optimization?

- Do not need to calculate the local sensitivity of each design variable
- Designer gets a view of entire design space
- Utilize the information collected from various sources and by different tools: CFD, semi-empirical, experiment, other studies
- Filter the noise intrinsic to numerical and experimental data
- Handle trade-offs and multi-criterion optimization

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# Characteristics of Global Optimization

- ***Scaling***: Requires More Data as the Number of Design Variables Increases
- ***Repeatability***: Same Data Can be Used When Conducting Comparison, Refinement, Similar Investigations
- Approaches in Global Optimization Methods:

  * ***Multi-Criterion Optimization***: Competing Objectives,

  * ***Multi-Level Optimization***: Refined Optimal Design Selections

  * ***Multi-Domain Optimization***: Adaptive Specification of Design Space

  * ***Multi-Point Optimization***: Identification of All Zones in Design Space Which Are Competitive

# OUTLINE OF THE TECHNIQUES

- Approximation
  - Polynomial-based RSM
  - NN- based RSM
  - Outliers analysis
- Design Of Experiment (DoE)
  - FCCD
  - OA
  - D-Optimality
- Optimization

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# What Is RSM?

- RSM '**A collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes and this provides an overall perspective of the system response within the design space**' Myers and Montgomery (1995)

- Often confused with the fitting procedure which is only a part of RSM procedure

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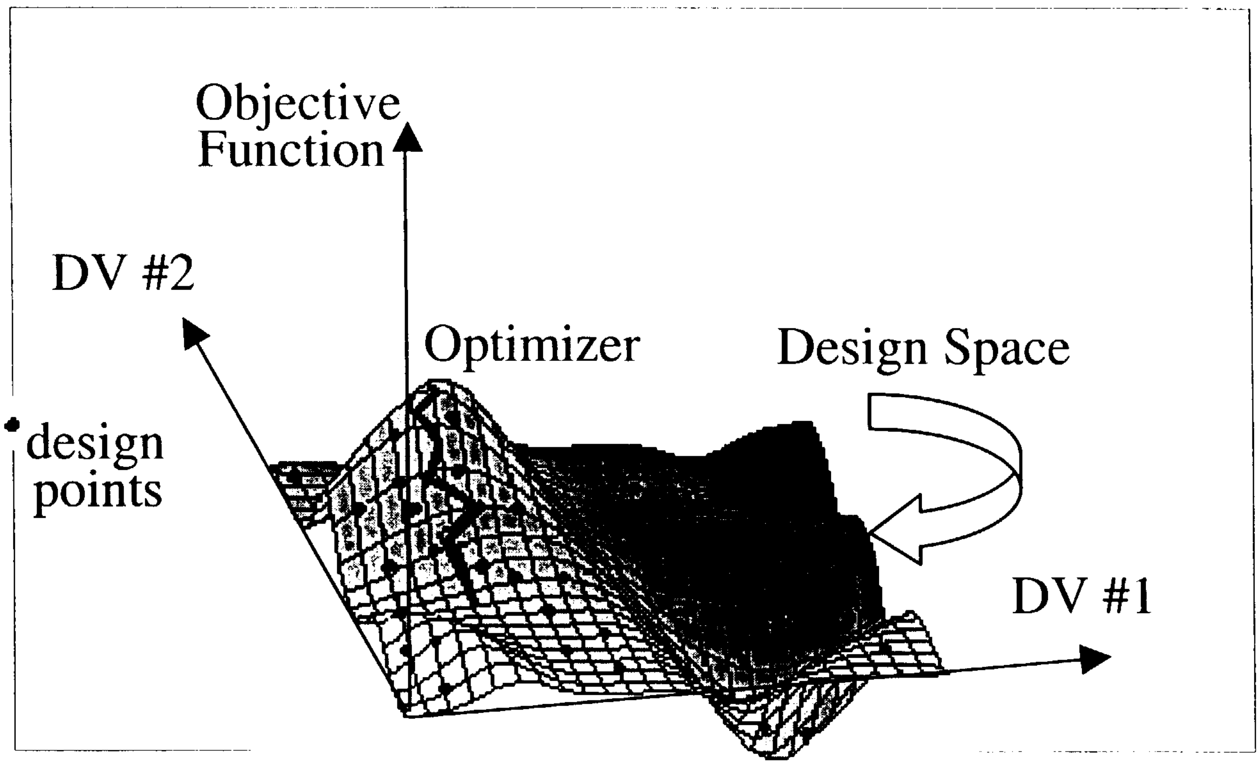
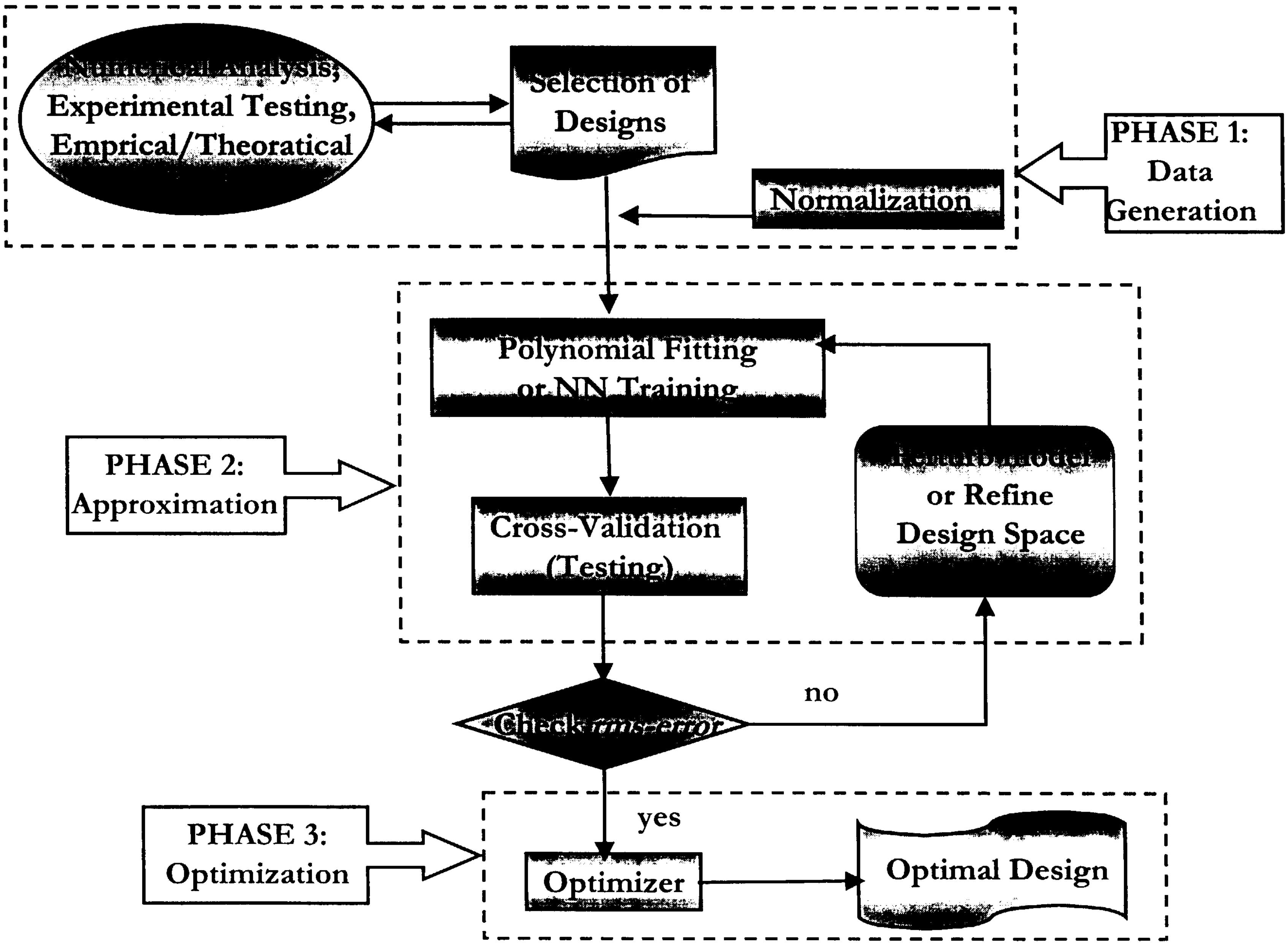
# How Does RSM Work

Replaces the objective and/or constraint functions by simple functions, often polynomials, which are fitted to the carefully selected design points.



# OVERALL RSM APPROACH



# POLYNOMIAL-BASED RSM

- **Polynomials of assumed order and unknown coefficients based on regression analysis**

- Number of coefficients to be evaluated depends on the order of polynomial and the number of design parameters involved

    - Second-order polynomial of N design variables has **$(N+1)(N+2)/2!$** coefficients (e.g. if $N=6 \Rightarrow$ # of coeff= 28)
    - Cubic model has **$(N+1)(N+2)(N+3)/3!$** coefficients (e.g. if $N=6 \Rightarrow$ # of coeff= 84)

- Polynomial models are constructed by **standard least-square regression**

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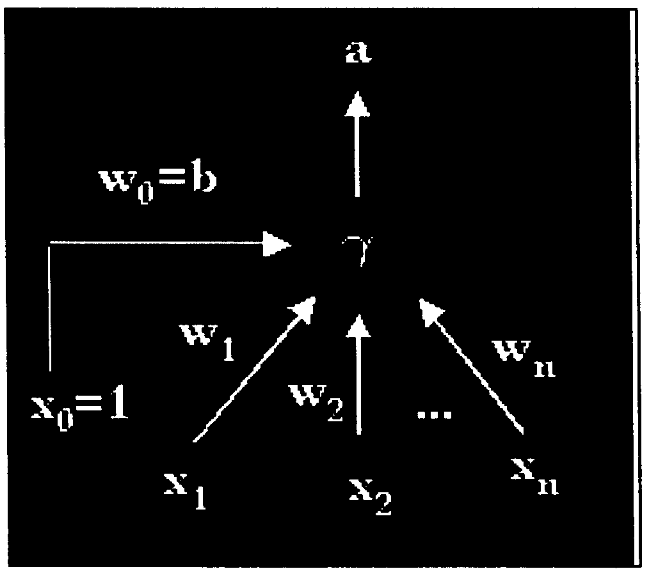
# Neural Networks

- Non-linear function approximators (with exceptions)
- Composed of simple computational units
- Connected together massively and in parallel
- Network established by adjusting the strength of connections between units (weights)




- $X=[x_0\ x_1\ x_2\ \ldots\ x_n]^T$: vector of scalar inputs
- $W=[w_0\ w_1\ w_2\ \ldots\ w_n]^T$: input weights
  $w_0$ : bias
- $\gamma$ : activation function
- a : scalar output

$$a = \gamma(W.X) = \gamma(\sum_{i=0}^{n} w_i x_i)$$

$\therefore$ A unit of NN sums up its total input and passes that sum through an activation function.

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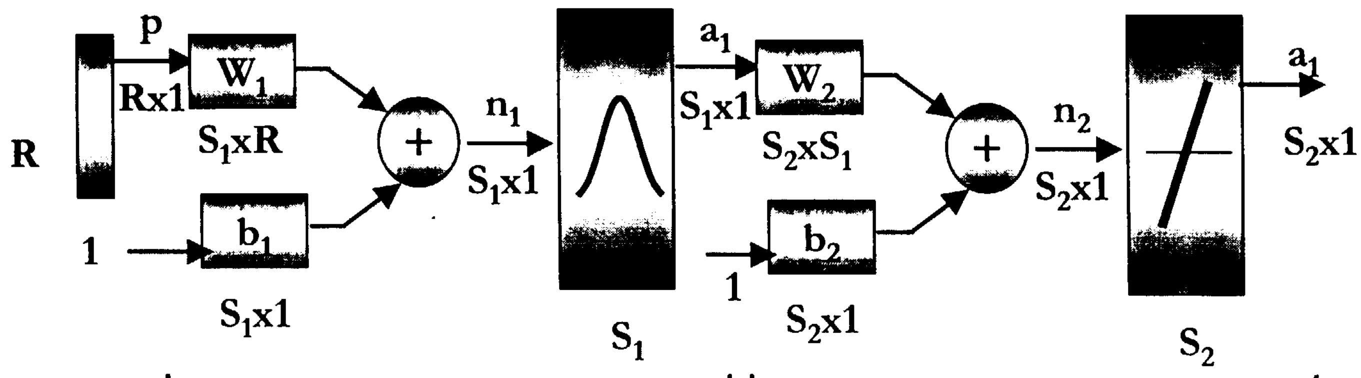
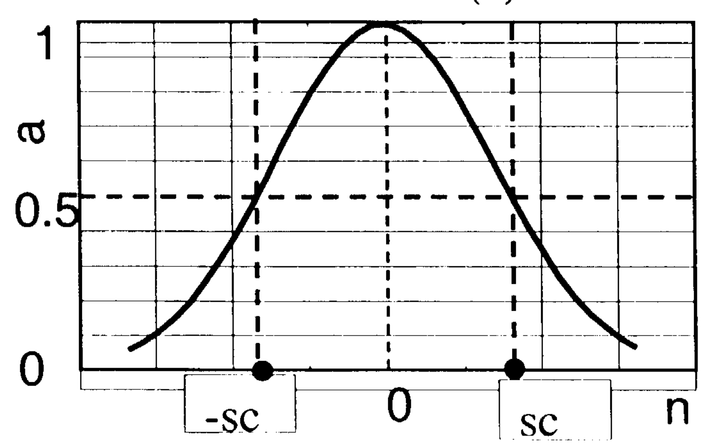
# Radial Basis Neural Network




*Radial Basis Activation Function*




$radbas(n) = e^{-n^2}$

$sc = 0.8326/b$

# WHY RBNN?

- **Back-propagation** NN's (BPNN) are the most commonly employed NN type in literature

- **RBNN's** are not as much in use as BPNN but they have two important advantages:
  - Training process of RBNN is a **linear problem** in terms of the weights and one can add/adjust neurons quite easily
  - Computations are relatively **cheap**

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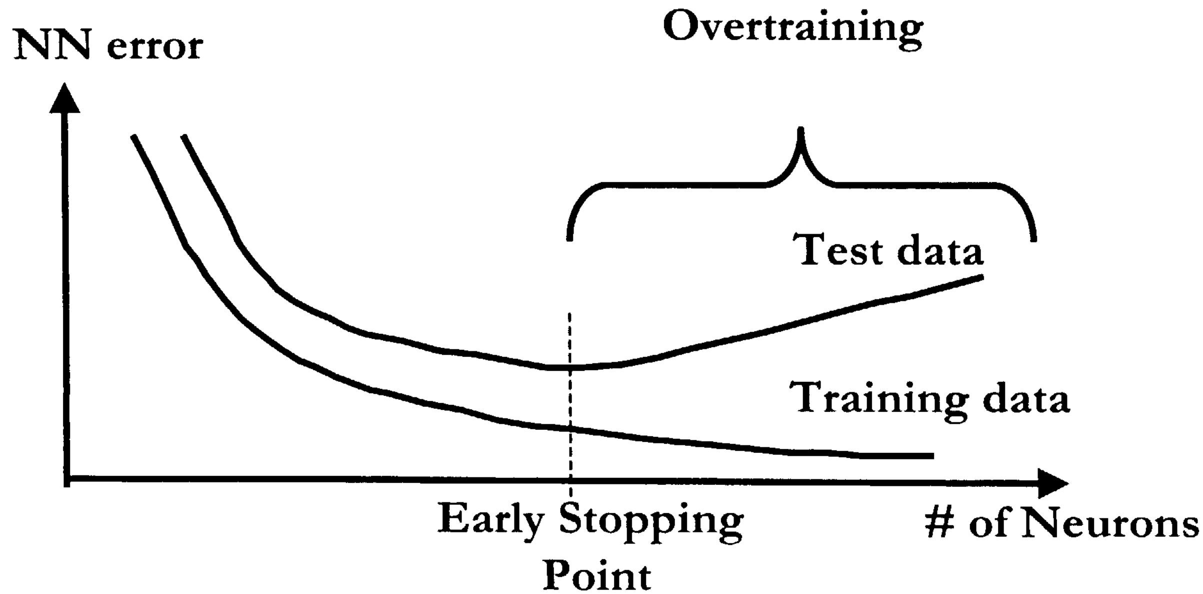
# PROCEDURE OF USING NN'S

- Collect training data for Input/Output
- Select NN architecture
- Train the weights of NN to minimize the error measure
- Generalization
  - Choosing not over-parameterized NN architecture
  - Testing or Cross Validation

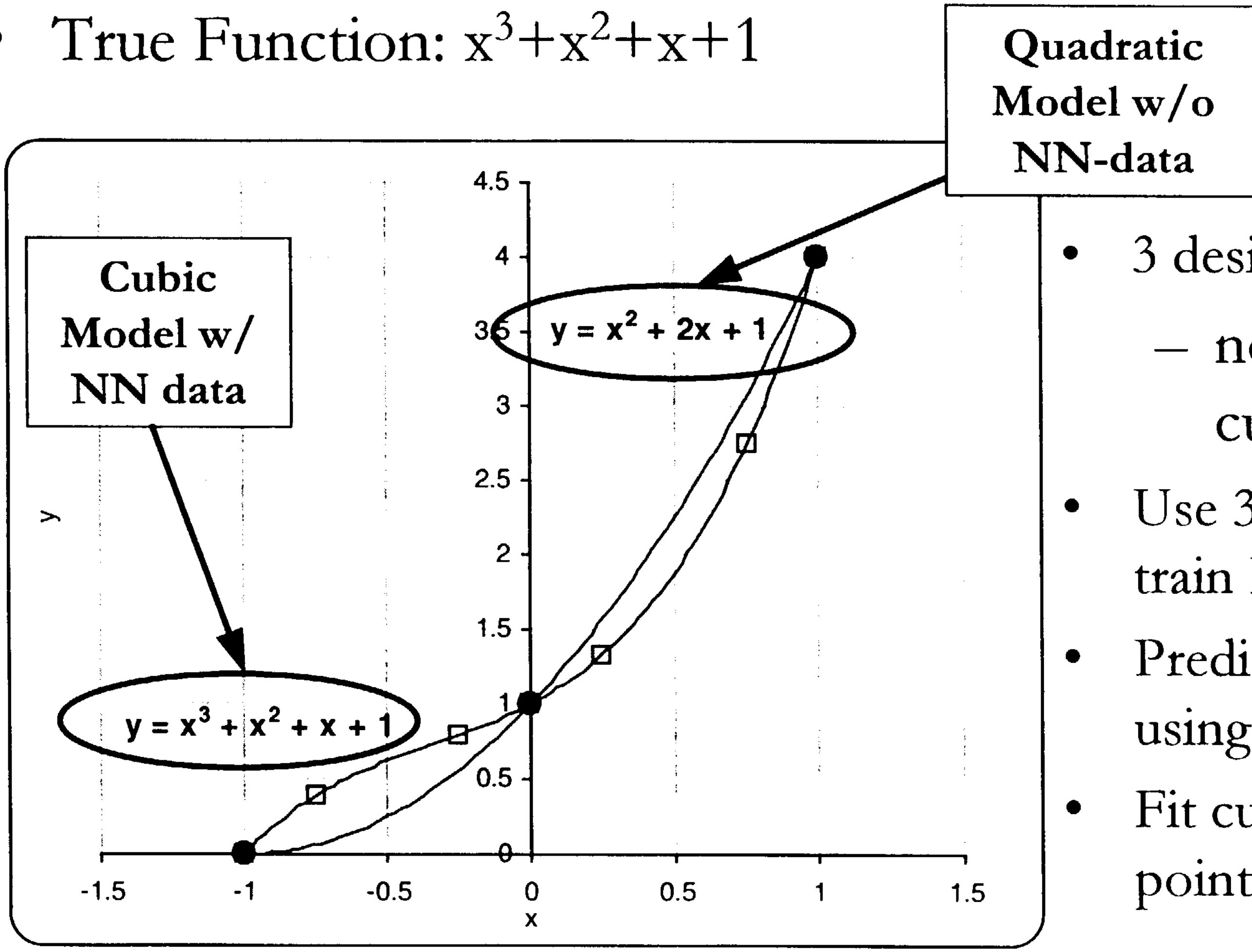


# RBNN-Enhanced RSM

- Use NN to generate additional data to feed the polynomial model
- **Improve the accuracy** of the response surface when there is insufficient amount of information available
  - Allows the optimization task to be done with **smaller number of CFD runs**
    - ⇒Reduces the cost of constructing response surface
    - ⇒Need to find ways to evaluate the fidelity of the NN data.

# EXAMPLE FOR NN-ENHANCED RSM

- True Function: $x^3+x^2+x+1$




- 3 design points available
  - not enough to fit cubic model
- Use 3 points available to train RBNN
- Predict 4 more points using RBNN
- Fit cubic model for 7 points

# DESIGN OF EXPERMENTS (DOE)

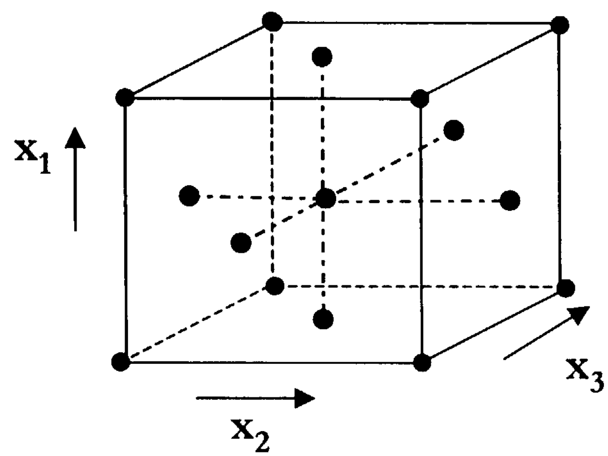
Statistical tools used to select the representation of the design space

- helps to minimize the effect of noise on the fitted polynomial
- improves efficiency and effectiveness of the construction of the response surface.

<u>**DOE tools:**</u>

- **Face centered composite design (FCCD)**
- **Orthogonal arrays (OA)**
- **D-Optimal design**, etc.

# FACE CENTERED COMPOSITE DESIGN (FCCD)




Yields ($2^N$+2N+1) points
N: the number of design variables

- For N=3 ⇒ # of design points= 15 (a design space composed of **8 corners** of the cube, **6 center of faces** and **the center of the cube)**

For N=6 ⇒ # of design points= 77

For N=11 ⇒# of design points= 2071

∴ More effective **when the # of design variables is modest**

- Widely used **for fitting second-order response surface**

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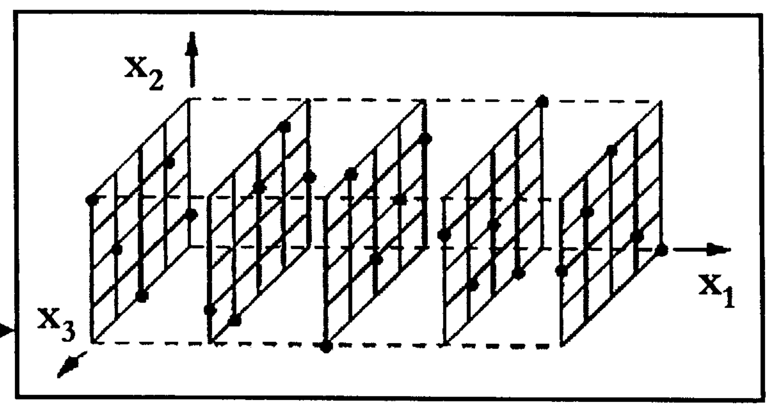
# Orthogonal Arrays (OA)

- Fractional factorial matrix that assures a balanced comparison of levels of any factor or interaction of factors

- Because the points are not necessarily at vertices, the orthogonal array can be more robust than FCCD

- OA can significantly reduce the number of experimental configurations

**Sample points of OA**



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# D-Optimal Design

- Minimize the generalized variance of the estimates, which is equivalent to maximizing the determinant of the moment matrix, M

$$|M| = \frac{|X^T X|}{n^{n_p}}$$

where ***X***: an $(n \times n_p)$ matrix of the levels of the independent variables

$n$: the number of observations

$n_p$: is the number of terms in the model

- Require specification of the properties of polynomial model in selecting the design points.

# OUTLIER ANALYSIS

- Helps to detect ***outliers*****, the points with excessive errors**, in the data sets that might adversely affect the accuracy of response surface.
- To check the existence of outliers, can employ **Iteratively Weighted Least Square (IRLS)** method to assess **weights for points with large residuals**.
- Weighting Formula:

$$weight = \begin{cases} \left[1 - \left(\dfrac{r/\sigma}{B}\right)\right], & if\ |r/\sigma| \le B \\ 0, & otherwise \end{cases}$$

r: residuals
σ: *rms-error*
B: tuning constant
(*1*<*B*<*3*) (Here B=1.9)

- If ***weight <0.01***, then the point is identified by IRLS as **an outlier**

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# OUTLIERS SUMMARY FOR 1ST VANE of Two-Stage Supersonic Turbine

**Original CFD Runs: 219**

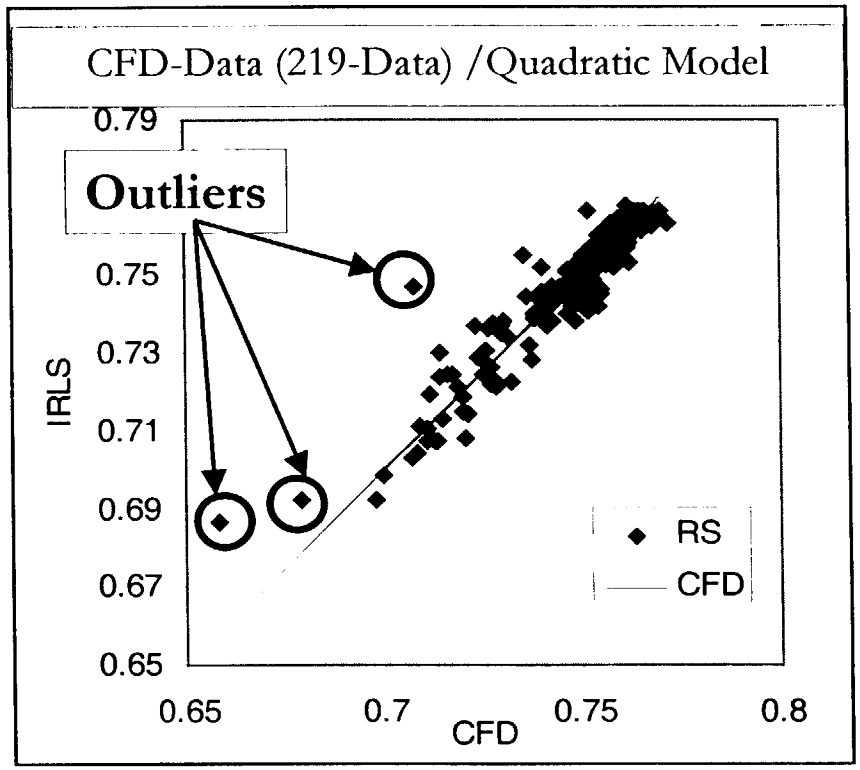
**Additional NN-Trained Data: 87**

**Quadratic and Cubic Models Are Constructed**

**Scope: Assess Presence of Outliers as NN Data Are Added.**

| | # of Outliers in CFD Data | # of Outliers in NN Data | Total # of Outliers | Total # of Data |
|---|---|---|---|---|
| CFD Data (Full Quadratic) | 17 | - | 17 | 219 |
| CFD+NN Data (Full Quadrati | 11 | 23 | 34 | 306 |
| CFD+NN Data (Full Cubic) | 3 | 14 | 17 | 306 |
| CFD+NN Data (Reduced Cubi | 5 | 13 | 18 | 306 |

# OUTLIERS ANALYSIS FOR 1$^{ST}$ VANE




**17 Outliers**

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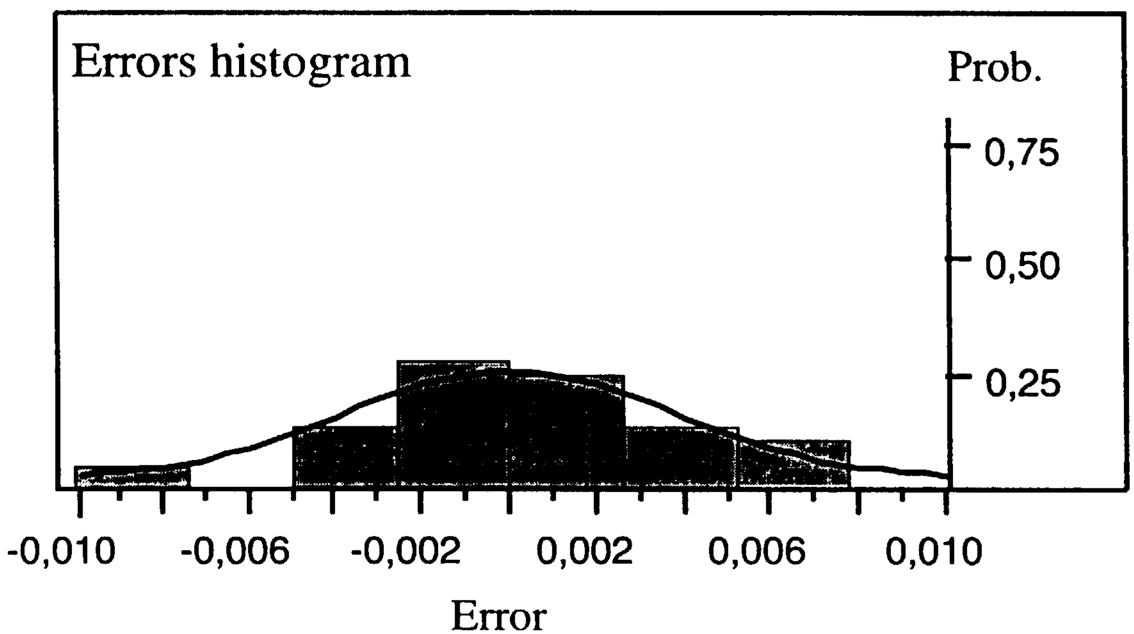
# Error Distributions:

## Turbulent Planar Diffuser Shape Optimization

**Distribution of response surface errors at sampling points and the corresponding normal distribution curve**

**Abnormal distributions can be attributed to outliers. No concern here.**



# OPTIMIZATION

## General Formulation

- The goal of an optimization problem is to find the combination of parameters (i.e., **design variables**) which optimize a given quantity (i.e., **objective function**), possibly subject to some restrictions (i.e., **constraints**) on the allowed parameter ranges.

- The general optimization problem may be stated mathematically as:

$$\textit{Minimize } \mathrm{f(x)}, \quad \mathrm{x}=(x_1, x_2, \ldots, x_N)^T$$
$$\text{subject to } h_i(x) = 0, \; i = 1, 2, \ldots, m$$
$$g_i(x) \geq 0, \; i = 1, 2, \ldots, n$$

- Optimization toolbox in Matlab and Excel Solver are often suitable

# Preliminary Design optimization of Supersonic Turbine

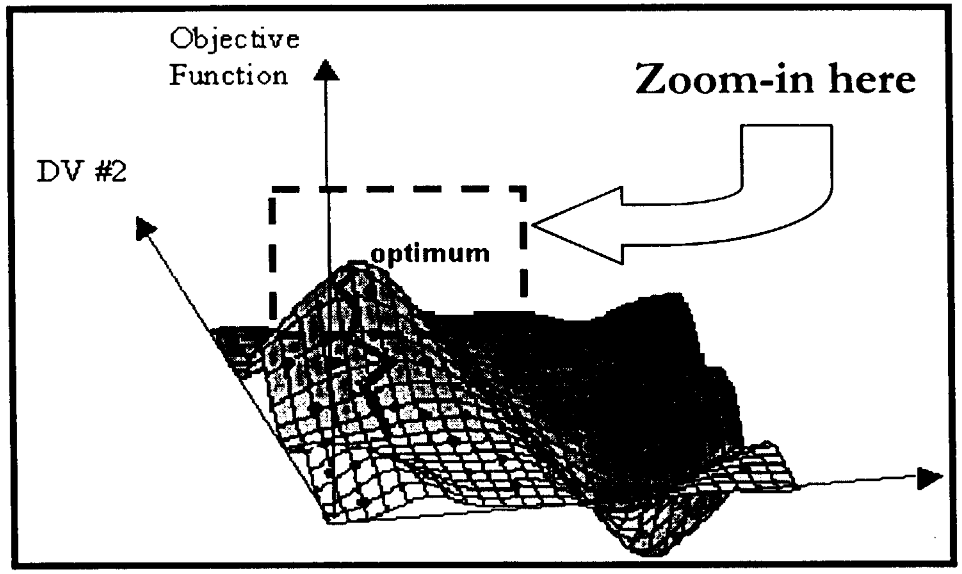
- Single-, 2- and 3-stage turbines have 6, 11 and 15 design parameters respectively

- Two objective functions for all of the cases:
  - Overall efficiency of the turbine, $\eta$.
  - Turbine weight, $\boldsymbol{W}$

- Two constraint functions for all of the cases :
  - A lumped inertia measure, $(AN)^2 = A_{ann}.RPM^2$
  - Speed at pitchline, $V_{pitch} = D\ .RPM$

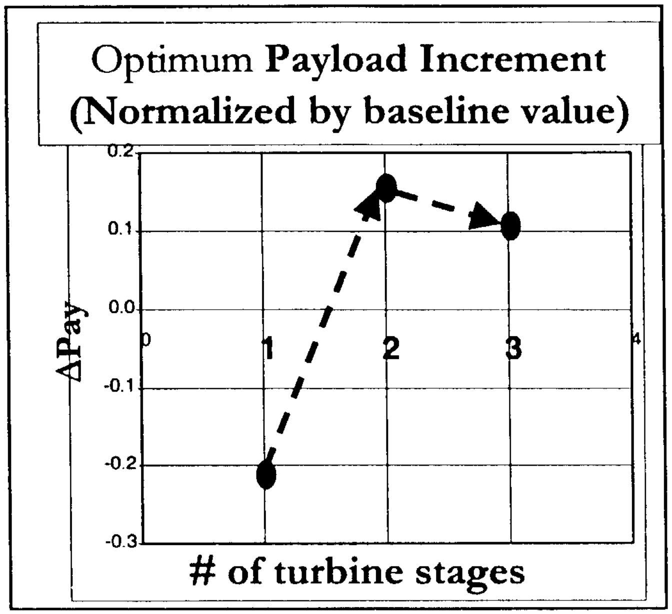
- Maximize $\eta$ and minimize $W$ simultaneously
  - OR maximize incremental payload (***Δpay***) which depends on $\eta$ and $W$

# MULTI-LEVEL APPROACH



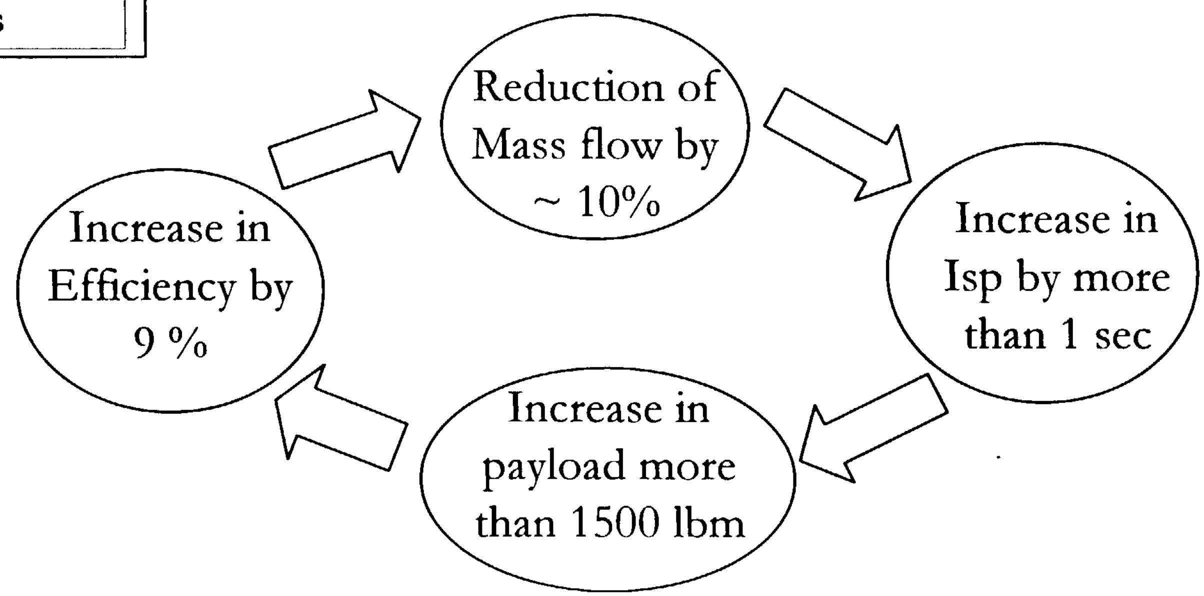

- Construct a global RS in the Original Design Space
- Identify optimum design with favorable performance
- Around the optimum, refine the design space
  - A refined design space of 1/5 size of the original design space with the center of the optimum design calculated from original design space
- Obtain the refined response surfaces repeating the RSM procedure

# PRELIMINARY DESIGN OPTIMIZATION OUTCOME




- ✔ The improvement in efficiency is offset by increased weight
- ✔ **2-Stage Turbine is the Optimum Configuration**
- ✔ Optimum configuration has a predicted **increase in $\eta$ of approximately 9 %** over the baseline

**Outcome:**



# Multi-Stage & Adaptive Domain Optimization: SUPERSONIC TURBINE: SHAPE DESIGN

- **Detailed shape design :**
  - The detailed shapes of the turbine vanes and blades, the final sizing and performance and clearance chosen
- Design Process - **Computational Fluid Dynamics (CFD), Neural Network** and **Polynomial-based RSM** employed
  - **CFD Analysis: Wildcat (quasi 3D) and Corsair (3D)**
    - Parallelized
    - Unsteady
    - Navier-Stokes
    - Moving grids
  - **Run time,** for e.g., **214 cases** for the first vane ~ **1.5 weeks** on one processor of and SGI (Origin 2000, Power Challenge, or Octane)
  - CFD runs done by Lisa Griffin and Dan Dorney

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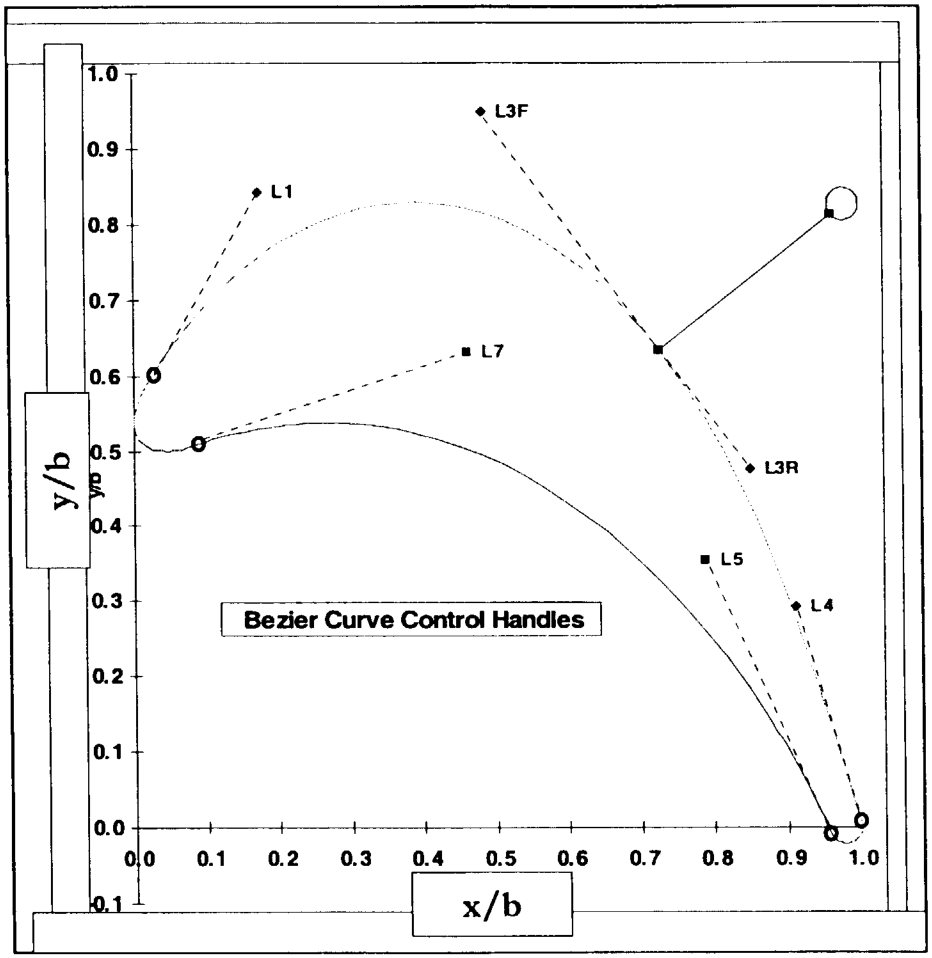
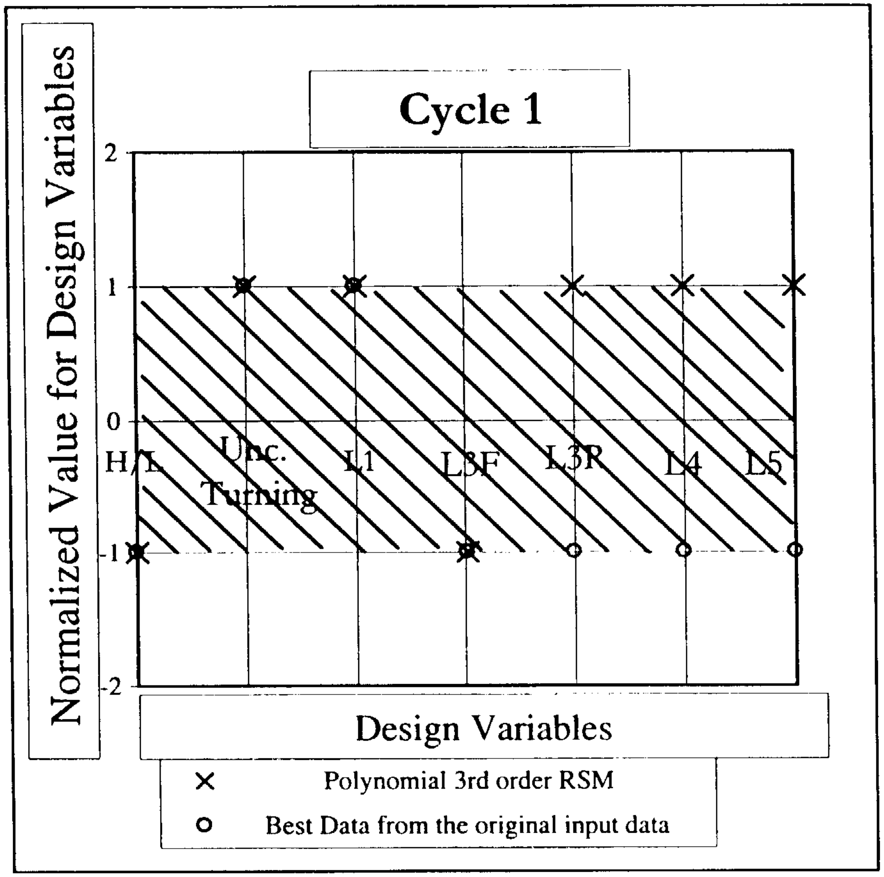
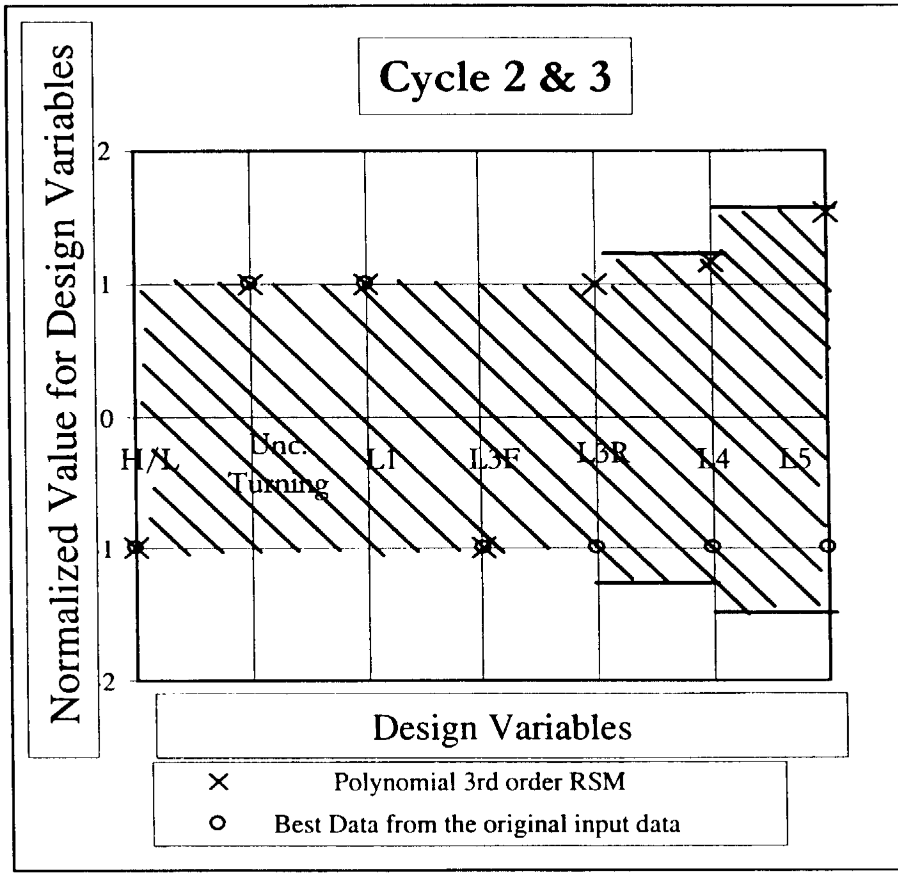
# 1st VANE DESIGN SCOPE




- **7 design parameters:**
  - LE pressure side height/axial chord
  - Uncovered turning
  - 5 Bezier curve control handles (*L1*, *L3F*, *L3R*, *L4*, and *L5*)
- Objective: **Maximize the stage efficiency**
- DOE: **FCCD+D-Optimal Desings (+OA)**
- **Reduced Cubic Models**

# 1st VANE SHAPE OPTIMIZATION




- Optimum design variables at the **limits of the design space**
- **Interactive nature of gobal optimization** helps to gain more insight and **revise the design scope** accordingly

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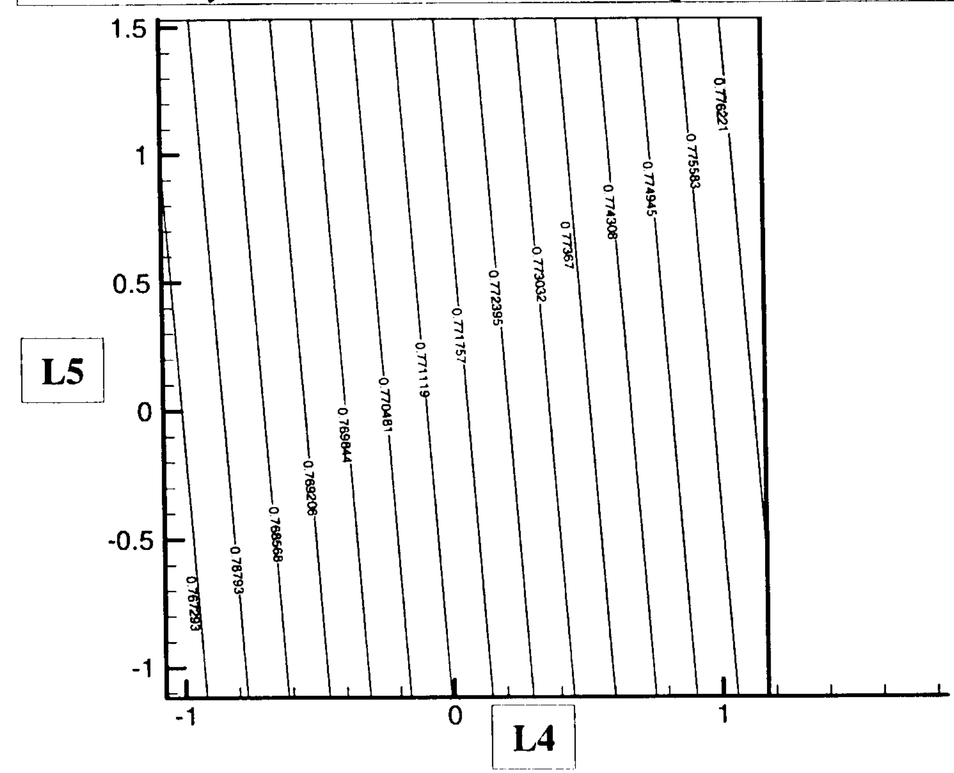
# Sensitivity Evaluation:
# 1st VANE RESULTS

- Response surface **flat with respect to L5** in region of highest efficiency: **can largely neglect it**.
- **L5** affects the **thickness of the airfoil**
- **Inspecting the influence of each design variable** enables to to probe alternative designs

Efficiency Contours around Highest Efficiency



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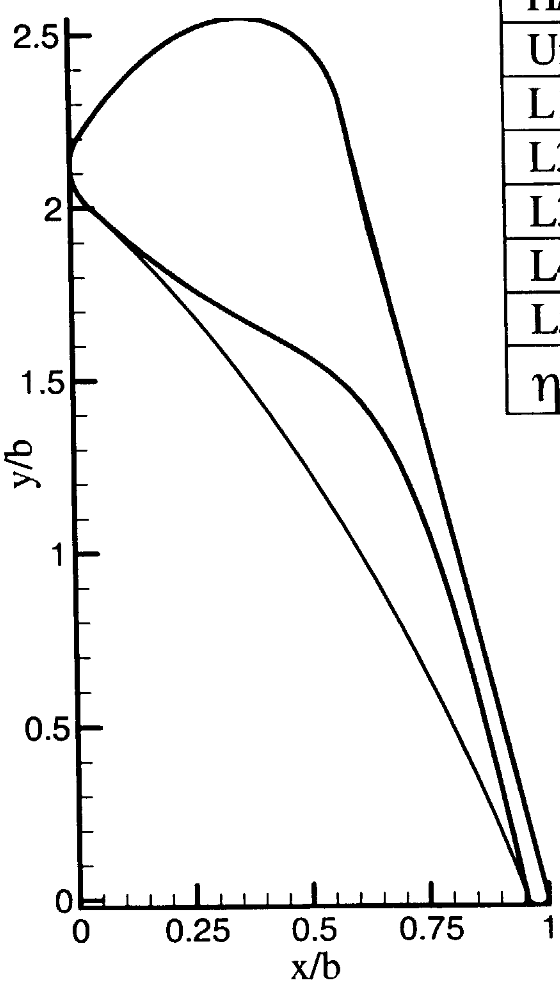
# OPTIMUM 1st VANE DESIGN

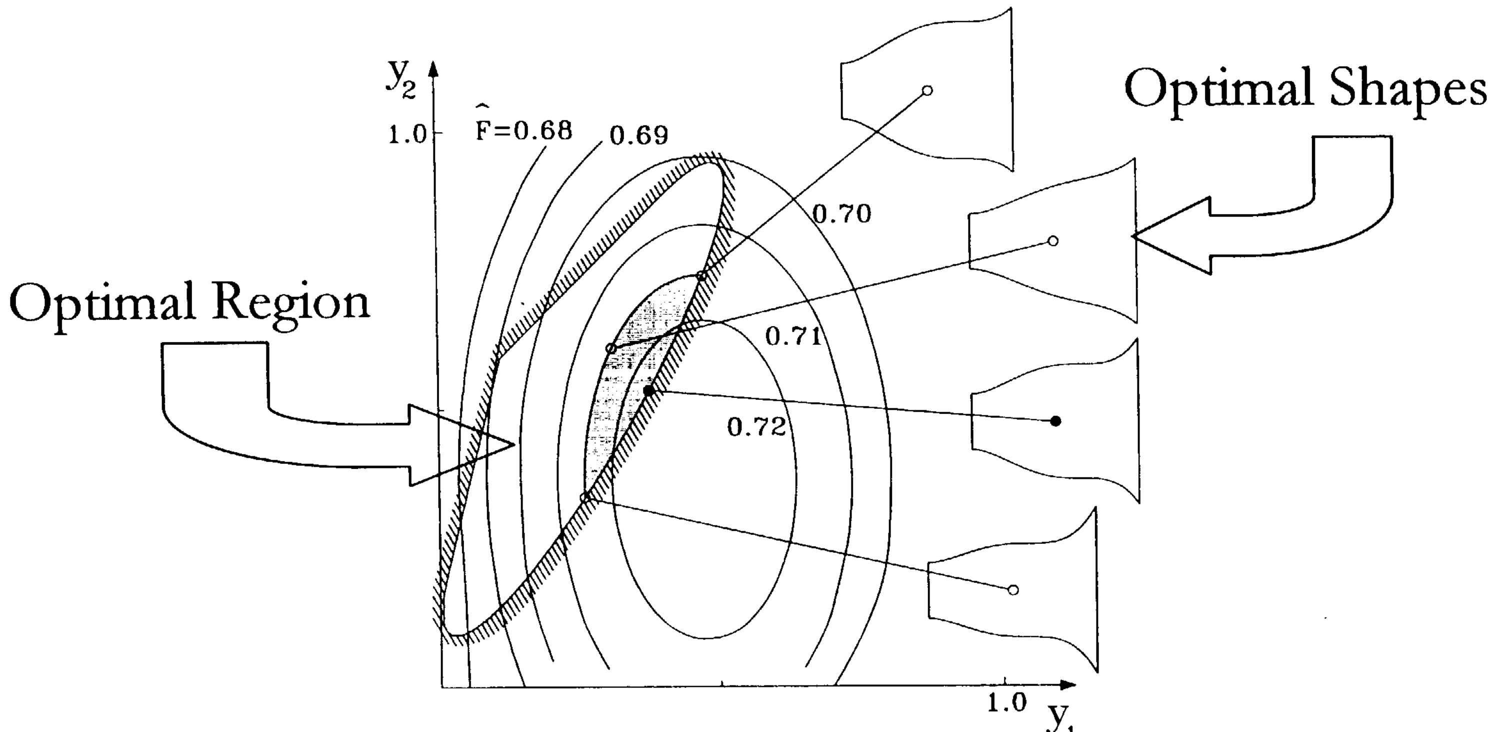



| Design Variable | Thinner Profile | Thicker Profile |
|---|---|---|
| H/L | 0.79 | 0.79 |
| Uncovered Turning | -0.20 | -0.20 |
| L1 | 2.00 | 2.00 |
| L3F | 0.44 | 0.44 |
| L3R | 0.25 | 2.00 |
| L4 | 0.05 | 1.35 |
| L5 | 0.13 | 2.50 |
| $\eta_{T-T}$ | **1.04** | **1.04** |

- Both shapes have comparable performance
- L3R and L4 do not affect the airfoil shape much but L5 does
- **Thicker profile** is selected for structural considerations

# Multi-Point Optimization: TURBULENT PLANAR DIFFUSER SHAPE OPTIMIZATION

## Contour Plot of Response Surface




- The hatched part of the feasible space comprises designs with performance within 1% of the optimal.

- **Multi-Optimum**: More than one design meet the goal.